

3.OA.1

Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that

 $a \times b$ means a groups of b objects each; however, because of the commutative property, students may also interpret 5×7 as the total number of objects in 7 groups of 5 objects each).

Essential Understandings

- Multiplication is repeated addition.
- Because multiplication is related to addition, order doesn't matter when you multiply, e.g. 3×4 is the same as 4×3 .

Common Misconceptions

Students get confused when thinking about the number of groups and the number in each group.

Students add the two numbers without thinking about multiplication as equal groups. For example, 5×4 represents 5 groups of 4 items but 5 + 4 represents a group of 5 items and a group of 4 items.

Academic Vocabulary/Language

- equal groups
- multiplication
- multiplication sentence
- factors
- multiply
- product
- array
- community property of multiplication
- repeated addition
- area models

Tier 2

- interpret
- solve
- identify

Learning Targets

I can explain why 5×7 is the same as 7×5 by using visual models to compare the two.

I can represent multiplication with equal groups, arrays, equal jumps on a number line, and with an area model.

I can relate repeated addition to representations of multiplication.

I can justify why an equation represents a multiplication situation.

I can interpret the meaning of the product and the factors of a multiplication equation.

- Students will identify the total number in all by multiplying the number of groups by the number of items in each group.
- Students will use physical models to show equal groups and connect them to drawings and equations.
- Students will explain the relationship between repeated addition and multiplication.
- Students will describe a problem situation that could be represented by a given multiplication equation.
- Students will represent and solve problems involving multiplication.

Sample Questions

- 1. Write a problem that can be solved using the multiplication equation 7×5 (e.g. Michael has 7 bags of apples. There are 5 apples in each bag. How many apples does Michael have altogether?)
- 2. How can I use a 8×3 array to determine the product of 3×8 ?
- 3. Show 6×5 in 2 different ways. You can show it with an array, equals groups, repeated addition, or jumps on a number line.
- 4. Write an expression that could be used to calculate the number of balls?







Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students develop an understanding of the meaning of multiplication by solving problems involving equal groups. Students use physical models, drawings, equations, number lines, and other visual representations to apply this understanding. Students will engage in mathematical discourse to explain their reasoning and compare it to the reasoning of others. Allow students to productively struggle to gain understanding of the concept.

Connections Across Standards

Multiply side lengths to find areas of rectangles (3.MD.7).

Represent the understanding of equal groups to create scaled graphs (3.MD.3).

Understand properties of multiplication and the relationship between multiplication and division (3.OA.5-6).

2.OA.4 (Prior Grade Standard)

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

4.OA.2 (Future Grade Standard)

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)



3.OA.2

Interpret whole number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a

number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

Essential Understandings

- There are 2 major types of division situations: fair sharing and repeated subtraction.
- Division is related to subtraction, so $56 \div 8$ can be solved by subtracting 8 until you reach zero or have less than 8 left.
- Division is related to multiplication.

Common Misconceptions

Students think that $15 \div 3$ is the same as $3 \div 15$

Students get confused about how to fair share. If the problem is $15 \div 3$, they might draw 3 groups but start by putting 15 in the first group rather than fair sharing the 15 out into the 3 groups to get 5 in each group.

Academic Vocabulary/Language

- divide
- dividend
- divisor
- division sentence
- quotient
- equal
- fair share
- array
- repeated subtraction

Tier 2

- describe
- solve
- determine

Learning Targets

I can determine the number of objects in each group when dividing a whole into equal groups.

I can determine the numbers of groups when the size of the group is known.

I can solve real world problems using division and justify my strategy.

I can represent division with models and drawings.

I can write an equation for a division situation and explain how it represents the situation.

- Students will determine the size of each group when the number of groups is known. (i.e. 18 cupcakes in 3 boxes. How many are in each box?)
- Students will determine the number of groups when the size of each group is known. (i.e. 18 cupcakes with 3 in each box. How many boxes are needed?)
- Students will solve a division problem with models, fair sharing, repeated subtraction, and physical models.

Sample Questions

- 1. Write a number expression that would explain how many pieces of candy 8 students would get if they share 56 pieces equally?
- 2. Robert cuts 16 feet of string into 4 equal pieces so he can share it with his three friends. How long is each piece of string?
- 3. Draw an array to show $45 \div 9$.
- 4. Write a story problem to match the equation $55 \div 11 = 5$.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students understand fairness and whether they are getting a "fair share" of something. Build on this innate understanding as you teach the concept of division. Provide examples for students to use both types of division strategies - repeated subtraction and fair shares. Relate fair shares to dealing a deck of cards: every player gets one card before any player gets two. Give students many opportunities to explore the relationship between multiplication and division. Division is taking a whole and dividing it into equal groups. Multiplication is counting those equal groups. Help students to interpret the "÷" symbol as meaning partitioning the total into equal groups or an equal number in each group.

When solving division word problems students should have the opportunity to explain their thinking verbally and engage in mathematical discourse to explain their reasoning and compare it to the reasoning of others. Students must be able to write an equation and explain how it connects to a problem or representation. Students should be able to answer "are we finding the numbers of groups?" (partitive) or "are we finding the group size?" (measurement division) each time they solve a division word problem. Students must be able to represent division using models, drawings, and manipulatives (counters, color tiles, base ten blocks). The use of counting strategies for multiplication and division problems is similar to the use of counting strategies for addition and subtraction problems. Students proficient with skip counting have a deeper conceptual understanding that groups of objects can be counted in the same way that single objects are counted (i.e. 4, 8, 12, 16).

Connections Across Standards

Multiply side lengths to find areas of rectangles (3.MD.7).

Represent the understanding of equal groups to create scaled graphs (3.MD.3).

Understand properties of multiplication and the relationship between multiplication and division (3.OA.5-6).

2.OA.1 (Prior Grade Standard)

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Table 1.

4.OA.2 (Future Grade Standard)

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)



3.OA.3

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and

equations with a symbol for the unknown number to represent the problem. See Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Essential Understandings

- Represent and solve word problems involving multiplication and division.
- Real-world mathematical situations can be represented using drawings and equations.

Common Misconceptions

Students may not know if the problem represents an addition situation or a multiplication situation

Students may not know if the problem represents a subtraction situation or a division situation

Academic Vocabulary/Language

- array
- equation
- variable
- multiplication
- division

Tier 2

- solve
- represent

Learning Targets

I can solve real world problems using multiplication and division.

I can construct meaning of multiplication and division problems up to 100 constructing models of equal groups, arrays, and measurement quantities.

I can compare my strategy for solving multiplication and division problems to others' strategies and explain similarities and differences between the two

- Students will represent a multiplication or division word problem with models, drawings, and equations.
- Students should engage in problem-solving with opportunities to discuss diverse representations of solutions.
- Students should have the opportunity to listen to other strategies, make connections, and develop an understanding of different problem solving situations through practice and conversations.

Sample Questions

- 1. If 24 plums are shared equally into 4 bags, how many plums will be in each bag?
- 2. How many different ways can you arrange 30 chairs in equal rows?
- 3. Each team has 5 players on it. There are 25 players in all. How many teams are playing in the game?
- 4. A group of 40 third grade students in the after school program are going on a field trip. The teacher chaperones decide to transport the students. Each car can hold 5 people. How many cars will they need to use?

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

There are many ways that students can represent problems to solve them. Provide students with a variety of problem solving tools as they work with multiplication and division word problems. Sets of counters, number lines, 100 charts and arrays/area models will aid students in solving problems involving multiplication and division. Allow students to model problems using these tools. They should represent the model used as a drawing or equation to find the solution.



This shows multiplication using grouping with 3 groups of 5 objects and can be written as 3×5 . This model can also represent 15 divided into 3 equal groups. Models like this help to show the relationship between multiplication and division. Students should be able to connect an equation to their models or drawings.

TABLE 2. COMMON MULTIPLICATION AND DIVISION SITUATIONS¹

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?	If 18 plums are to be packed 6 to a bag, then how many bags are needed?
	Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS ² , AREA ³	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
	Area example. What is the area of a 3 cm by 6 cm rectangle?	Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?
	Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Connections Across Standards

Multiply side lengths to find areas of rectangles (3.MD.7).

Represent the understanding of equal groups to create scaled graphs (3.MD.3).

Understand properties of multiplication and the relationship between multiplication and division (3.OA.5-6).

2.OA.4 (Prior Grade Standard)

Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

4.NBT.5 (Future Grade Standard)

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



3.OA.4

Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation

true in each of the equations $8 \times \square = 48$; $5 = \square \div 3$; $6 \times 6 = \square$.

Essential Understandings

- Represent and solve problems involving multiplication and division
- Understand that the unknown in a problem can occur in any position within the equation and must make that equation true.

Common Misconceptions

Students think a symbol (? or \square or a) is always the place for the answer. This is especially true when the problem is written

as $6 \times 2 = \square \times 3$.

Students do not always use their understanding of the relationship of multiplication and division to solve equations. For example, if 5×6 is 30, then $30 \div 5 = \square$ must have an answer of 6.

Academic Vocabulary/Language

- factor
- product
- multiplication
- dividend
- divisor
- quotient
- variable
- equation

Tier 2

- solve
- determine
- represent

Learning Targets

I can solve real world problems using multiplication and division.

I can determine the unknown number in a multiplication equation when there is a variable (missing number) in the equation.

I can determine the unknown number in a division equation when there is a variable (missing number) in the equation.

I can explain how the relationship between the multiplication and division can be used when solving problems with an unknown variable.

- Students will determine the unknown number in multiplication and division problems.
- Students will solve real world problems using multiplication and division.
- Students can write an equation to a problem or situation (with and without the unknown).

Sample Questions

1. Write the missing numbers to make each equation true

$$? \times 6 = 36$$
 $16 = 4 \times \square$ $\square \div 7 = 8$

- 2. Beth has 4 boxes of books with "x" in each box. If Beth has 24 books, how many books are in each box?
- 3. Jennifer bundled up all of her old crayons into bags of equal groups. She had 72 crayons and was able to make 8 bags. How many crayons were in each group? Solve the problem and write an equation to represent the unknown.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students can use known multiplication facts to determine the unknown in a multiplication or division problem. Have them write a multiplication or division equation and the related multiplication or division equation. For example, to determine the unknown whole number in $27 \div \Box = 3$, students should use knowledge of the related multiplication fact of $3 \times 9 = 27$. They should ask themselves questions such as, "How many 3s are in 27?" or "3 times what number is 27?" Have them justify their thinking with models or drawings. It is also important that students can connect an equation to a problem or situation and see the relationship with the unknown number in an equation being in any position $(5 = 50 \div ?)$, $(n \times 5 = 50)$, and $(5 \times 10 = 50)$.

Connections Across Standards

Multiply side lengths to find areas of rectangles (3.MD.7).

Represent the understanding of equal groups to create scaled graphs (3.MD.3).

Understand properties of multiplication and the relationship between multiplication and division (3.OA.5-6).

2.OA.1 (Prior Grade Standard)

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Table 1.

4.OA.2 (Future Grade Standard)

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the standard.)



3.OA.5

Apply properties of operations as strategies to multiply and divide. For example, if $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known (Commutative Property of Multiplication); 3

 \times 5 \times 2 can be found by 3 \times 5 = 15, then 15 \times 2 = 30, or by 5 \times 2 = 10, then 3 \times 10 = 30 (Associative Property of Multiplication); knowing that 8 \times 5 = 40 and 8 \times 2 = 16, one can find 8 \times 7 as 8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56 (Distributive Property). Students need not use formal terms for these properties.

Essential Understandings

- The order of numbers in multiplication does not change the product.
- Numbers can be regrouped in a multiplication problem without changing the product.
- In multiplication, one factor can be decomposed into parts; each part is multiplied separately by the other factor, then the results are added.

Common Misconceptions

Students may not know that 5×2 is the same amount as 2×5 . Students may have difficulty seeing that an array can be broken apart and still represent the same whole. For example, 7×5 is the same as $2 \times 5 + 5 \times 5$.

Academic Vocabulary/Language

- commutative property
- associative property
- distributive property

Tier 2

- apply
- explain
- relate

Learning Targets

I can explain the properties of multiplication and division and how they relate to each other.

I can represent the properties of multiplication and division and how they relate to each other.

I can justify my strategy for grouping numbers together to solve a problem using models or numbers.

- Students will be able to explain and represent the commutative, associative, and distributive properties.
- Students can create physical models and drawings to prove that properties are true.
- Students can apply properties to recall basic facts or multiply with multiples of 10 (e.g. 3×50 can be thought of as $3 \times 5 \times 10$).

Sample Questions

1. Do the two arrays of circles show the commutative property? Explain how you know.

0 0 0 0 0 0	0 0
000000	0 0
	0 0
	0 0
	0 0
	0 0

- 2. Kelsey says that to multiply 17×5 , she first multiplies 10×5 . What must she do next to get the correct answer to 17×5 ?
- 3. Mary says that she can multiply $3 \times 5 \times 2$ more easily if she multiplies the 5×2 first and then multiplies that answer by 3. Explain why this should still give the correct answer.
- 4. Anthony knows 4×8 but doesn't know 4×16 . How could Anthony use 4×8 to find 4×16 ?

Students need not use formal terms for these properties.

Table 3: The Properties of Operation

Here *a, b, and c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system and the complex number system.

Associative Property of Addition (a + b) + c = a + (b + c)

Commutative Property of Addition a + b = b + a

Additive Identity Property of 0 a + 0 = 0 + a = a

Associative Property of Multiplication $(a \times b) \times c = a \times (b \times c)$

Commutative Property of Multiplication $a \times b = b \times a$

Multiplicative Identity Property of 1 $a \times 1 = 1 \times a = a$

Distributive Property of Multiplication over Addition $a \times (b + c) = a \times b + a \times c$

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students need to apply properties of operations (commutative, associative and distributive) as strategies to multiply and divide. Applying the concept involved is more important than students knowing the formal name of the property. Understanding of the commutative property of multiplication is developed through the use of models as basic multiplication facts are learned. For example, the product of multiplying 3×5 is the same as the product of multiplying 5×3 . The array can be turned from 3 groups of 5 to 5 groups of 3 and the total is still 15.

To find the product of three numbers, students can use what they know about the product of two of the factors and multiply this by the third factor. For example, to multiply $5 \times 7 \times 2$, students know that 5×2 is 10. Then, they can use mental math to find the product of 10×7 . Allow students to use their own strategies and share with the class when applying the associative property of multiplication. Relate this idea to addition. Order doesn't matter when you add or multiply, so just like they may try to make tens when solving an equation like 3 + 5 + 7 + 6 + 5, they might look for facts they know when solving an equation like $3 \times 5 \times 2$.

Splitting arrays can help students understand the distributive property. They can use a known fact to learn other facts that may cause difficulty. For example, students can split a 6×9 array into 6 groups of 5 and 6 groups of 4; then, add the sums of the groups.

This is also a good time to expose students to the Multiplicative Property of Zero by modeling, for example, what 3 groups of zero looks like or zero groups of 3. Students should also be exposed to the Multiplicative Identity Property of One by modeling, for example, 1 group of 5 and 5

groups of 1.

Properties should be identified correctly. Created names for properties (e.g. "flip flop") should not be used to describe the property.

Connections Across Standards

Represent and solve problems involving multiplication and division (3.OA.1-4).

Solve two-step word problems using the four operations (3.OA.8).

Multiply one-digit whole numbers by multiples of ten (3.NBT.3).

Relate area to the operations of multiplication (3.MD.7).

2.NBT.5 (Prior Grade Standard)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.OA.3 (Future Grade Standard)

Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.



3.OA.6

Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Essential Understanding

• Understand that the inverse, or opposite of division is multiplication, therefore the answer to $24 \div 8$ can be found by solving $8 \times \square = 24$.

Common Misconceptions

Students do not understand part to whole relationships.
Students do not understand the inverse relationship between multiplication and division.

Academic Vocabulary/Language

- dividend
- divisor
- quotient
- factor
- relationship
- inverse

Tier 2

explain

Learning Targets

I can identify the properties of multiplication and division and explain how they relate to each other. I can apply the properties of multiplication to find an unknown in a division equation and explain the logic for using this strategy.

- Students can explain the relationship between multiplication and division.
- Students can solve for the unknown in a division equation.
- Students can find a factor by dividing or taking away equal groups.
- Students can turn a division problem into a multiplication problem with an unknown factor.

Sample Questions

- 1. Paula says she solves the problem $56 \div 8$ by solving the related multiplication fact. What is the related multiplication fact?
- 2. Erica says that if you know $2 \times 6 = 12$, then you know what n equals in $12 \div n = 6$. Why is she correct?
- 3. Use the numbers 4, 9, and 36 to write a multiplication story. Write a related division story.
- 4. Rob knows that $9 \times 3 = 27$. How can he use that fact to find the answer to the following problem? 27 books are divided into 9 boxes. How many books are in each box? Write a division equation and explain your reasoning.
- 5. Kaylee had to solve 72 ÷ 9 by finding the number that makes 72 when multiplied by 9. What strategy can Kaylee use to solve this problem? Explain your reasoning.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Just like students are often more comfortable with addition then subtraction, students often feel that multiplication is "easier" than division. Use this to help students see that because of the inverse relationship between multiplication and division, multiplication facts can be used to solve division problems. They both involve a number of groups, a number in each group, and a total number. Students should have a variety of experiences looking at multiplication and division "fact families" or "number bonds", those part-part-whole relationships that exist between numbers. For example, they should know from experience that $3 \times 7 = 21$, so $21 \div 3$ must equal 7 because 3 and 7 and 21 are bonded together.

Connections Across Standards

Represent and solve problems involving multiplication and division (3.OA.1-4).

Solve two-step word problems using the four operations (3.OA.8).

Multiply one-digit whole numbers by multiples of ten (3.NBT.3).

Relate area to the operations of multiplication (3.MD.7).

2.NBT.5 (Prior Grade Standard)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT.6 (Future Grade Standard)

Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



3.OA.7

Fluently ^G multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 40$

8, or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one digit numbers

Essential Understandings

- Fluency means being efficient, accurate, and flexible with strategies.
- There is an inverse relationship between multiplication and division, so if a student knows the multiplication facts from memory then they also know the division facts from memory.

Common Misconceptions

The student may know the commutative property of multiplication but fail to apply it to simplify the "work" of multiplication. For example, the student states that $9 \times 4 = 36$ with relative ease, but struggles to find the product of 4×9 . The student sees multiplication and division as discrete and separate operations. For example, the

The student sees multiplication and division as discrete and separate operations. For example, the student has reasonable facility with multiplication facts but cannot master division facts. He may know that $6 \times 7 = 42$ but fails to realize that this fact also tells him that $42 \div 7 = 6$.

The student may know procedures for dividing but has no idea how to check the reasonableness of his answers.

Academic Vocabulary/Language

- inverse operation
- fact family
- related facts
- product
- quotient
- fluently
- efficiently
- divisor
- factor
- dividend

Learning Targets

I can efficiently multiply any two numbers that result in a product within 100.

I can efficiently divide whole numbers with a divisor within 100 that results in a whole number quotient.

I can strategically recall from memory the product of any two one-digit numbers.

I can justify the strategies I used to solve multiplication and division problems within 100.

I can verify the reasonableness of my answer by using known products of two one digit numbers.

- Students can efficiently multiply and divide numbers with a solution within 100.
- Students can recall facts from memory to solve multiplication and division problems.
- Students can use various strategies to develop mental math skills that can be applied to larger numbers.

Sample Questions

- 1. Amier thinks that $30 \div 6 = 5$. Is he correct? Use what you have learned about multiplication and division to support your thinking.
- 2. How are the following three problems related? $48 \div 12 = 24 \div 6 = 12 \div 3$
- 3. Maya's teachers told her that if she knows her multiplication facts for 8 then she also knows her multiplication facts for 4. Maya doesn't understand how she would know her multiplication facts for 4. Explain the relationships between the 8's and 4's facts.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students are expected to know from memory all multiplication facts of two one digit numbers through 9×9 by the end of the school year. This should happen only after a solid understanding of the meaning of multiplication is in place. The use of daily Number Talks can help to build this understanding. These Number Talks can help students to explore number relationships and look for patterns. These Number Talks also promote efficient mental strategies for multiplication and encourage a discussion of why some strategies are more efficient than others. An understanding of the Commutative Property of Multiplication can also help with the students' memorization of facts. Students may need to explicitly be shown that if they know 4×6 then they know 6×4 . The inverse relationship between multiplication and division means that once students know the multiplication facts, they know the corresponding division facts. Again, this may be something that students need to be explicitly told.

Students' ability to recall facts should be assessed in various ways. Fact recall can be assessed through classroom discourse, interviews, observations, observation of strategies when solving problems, and other alternative methods.

Connections Across Standards

Use all operations and algebraic thinking standards for Grade 3 (3.OA.1-6, 8-9).

Relate area to the operations of multiplication and addition (3.MD.7).

2.NBT.5 (Prior Grade Standard)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT.6 (Future Grade Standard)

Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



3.OA.8

Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the

reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.

Essential Understandings

- Represent and solve problems involving the four operations.
- The unknown in a problem can be represented with a symbol.
- Problems may have more than one step needed in order to find a solution.
- Rounding can be used to assess the reasonableness of answers.

Common Misconceptions

Students may work through a problem and find an answer but not stop to assess if this answer is reasonable

Students may not understand that just because \square represents 8 in one problem, it may not represent 8 in another problem.

Students may only complete step one in a problem that requires two steps.

Academic Vocabulary/Language

- variable
- evaluate
- equation
- estimate
- rounding
- pattern

Tier 2

- solve
- represent
- identify
- explain

Learning Targets

I can apply what I know about addition, subtraction, multiplication, and division to solve two-step word problems. I can apply what I know about addition, subtraction, multiplication, and division to solve two-step word problems with one unknown number.

I can determine if the answer to a two-step problem is reasonable by using mental math, estimation, and rounding to justify my logic.

- Students will create a plan for solving two step word problems and use estimation to determine the reasonableness of the answer.
- Students can write an equation using a letter for the unknown number.
- Students can identify multiple strategies to provide a solution to a two-step word problem.
- Students can determine if a solution to a two-step problem is reasonable by using mental math, estimation, and rounding.

Sample Questions

- 1. 78 39 = 39. Use mental math to make sense of this equation and explain your thinking.
- 2. You have been saving for a new remote control car that costs \$35. You have \$11. How much do you still need to save to buy the remote control car? Explain the strategy used to find the solution.
- 3. Harley bought 6 hotdogs and 2 hamburgers. He spent \$5.00. The hotdogs cost \$.50 each. How much did one hamburger cost?
- 4. Aaliyah needs 90 cupcakes for a birthday party. She has 29 chocolate cupcakes and 12 vanilla cupcakes. How many more cupcakes does she need?
- 5. Meagan has been saving up her money to buy a new art set. She saved \$15.00 last month. The art set cost \$50.00. Write an equation that could be used to figure out how much money Meagan needs to earn this month to buy her art set. Use the letter *m* to represent the amount of money she needs to earn this month.
- 6. There are 61 third-grade students in Ben's school. 14 of them are in the gym. How many are left in their classrooms? If each classroom holds about 20 students, how many classrooms are likely being used?

NOTE: This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

There are many ways that students can represent problems to solve them. Teachers should encourage diverse representations of strategies with the opportunity for students to verbally share and discuss strategies with peers to help students develop an understanding of the relationships within a problem. Through discussions, students can develop a sense of reasonableness of their answers. Encourage students to represent the problem situation in a drawing or with counters or blocks. Students should determine the reasonableness of the solution to all problems using mental computations and estimation strategies. As students show developing proficiency with two-steps problems, teachers can begin to create situations using any of the four operations.

Students gain a full understanding of which operation to use in any given situation through contextual problems. Number skills and concepts are developed as students solve problems. Problems should be presented on a regular basis as students work with numbers and computations. Researchers and mathematics educators advise against providing "key words" for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through the problem and the meaning of the answer before attempting to solve it.

Begin to use a letter to represent an unknown in an equation to prepare students for their future work with algebra.

Connections Across Standards

Use multiplication and division within 100 in word problems involving equal groups, arrays, and measurement quantities (3.OA.3).

2.NBT.5 (Prior Grade Standard)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.OA.3 (Future Grade Standard)

Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.



3.OA.9

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times

a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Essential Understandings

- Identify patterns in the addition table or multiplication table.
- Use their understanding of these patterns to help them solve problems.

Common Misconceptions

The student can state and give examples of properties of addition and multiplication but does not apply them to solve problems. Students may not use their understanding of patterns within addition, for example, to assess the reasonableness of their answer. For example, 6×2 cannot be 13 because when I skip count by 2 I always land on an even number. 13 is not an even number.

Academic Vocabulary/Language

- pattern
- commutative property
- associative property
- distributive property

Tier 2

- identify
- explain

Learning Targets

I can identify arithmetic patterns, especially in the addition and multiplication tables.

I can explain and justify the arithmetic patterns by applying the properties of operations when solving problems.

I can give examples of arithmetic patterns and explain my reasoning.

- Students can describe patterns in addition and multiplication tables.
- Students can explain patterns when adjusting addends. (i.e. 46 + 88 is the same as 44 + 90)
- Students can explain patterns of addition for example (even + even = even, odd + odd = even, and even + odd = odd).
- Students can determine that two addends less than 50 have a sum less than 100.
- Students can explain patterns in multiplication for example (even \times even = even, odd \times odd = odd, and odd \times even = even.

Sample Questions

- 1. Explain the reason why, when you add a number to itself, the answer is always even?
- 2. You are given two numbers whose difference is 8. If the one number is increased by 5, what needs to happen to the other number to have the difference remain 8?
- 3. Explain why multiples of 6 are always even and divisible by three.
- 4. Describe the pattern of answers whenever a number is divided by 10.
- 5. Candace says 9×4 is the same as doubling 9×2 . She says it also works when you multiply other numbers by 4. Explain why you agree or disagree with Candace?
- 6. Have students describe the relationships between multiplying by 2, multiplying by 4, and multiplying by 8. Have students make connections to solutions in order to think about how multiplies of given numbers are related. (This task can be done as a warm-up activity to create an opportunity for students to think about patterns that surface through the repeated addition of these equal-size groups and explore why the patterns exist. On day 1 chart the multiples of 2. On day 2 chart the multiples of 4. On day 3 have students compare the two charts and identify how the multiples are related. On day 4 chart the multiples of 8. Engage students in a discussion comparing the three charts to identify how the multiples are related.)

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students are to identify arithmetic patterns and explain them using properties of operations. They can explore patterns by determining likenesses, differences and changes. Use patterns in addition and multiplication tables to highlight numerical relationships and to connect properties of operations to arithmetic patterns. We can use patterns to solve problems, make calculations, or recall basic facts. It is important that teachers record equations intentionally so that patterns can be observed and discussed. These patterns should be discovered through investigation.

Connections Across Standards

Use multiplication and division within 100 in word problems involving equal groups, arrays, and measurement quantities (3.OA.3).

2.OA.2 (Prior Grade Standard)

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies.

4.OA.5 (Future Grade Standard)

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.



3.NBT.1

Use place value understanding to round whole numbers to the nearest 10 or 100.

Essential Understandings

- Rounding helps solve problems mentally and assess the reasonableness of an answer.
- Rounding helps to estimate.

Common Misconceptions

The use of terms like "round up" and "round down" confuses many students. For example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding "down". The number 32 should be rounded to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding.

Academic Vocabulary/Language

- round
- estimate
- nearest ten
- nearest hundred

Tier 2

solve

Learning Targets

I can round whole numbers to the nearest 10 and explain my reasoning.

I can round whole numbers to the nearest 100 and explain my reasoning.

I can apply the properties of rounding when solving problems and justify my reasoning.

- Students can round whole numbers to the nearest 10 and 100 and explain their reasoning.
- Students can use place value strategies (number line, near benchmark, friendly, or compatible numbers) to round to the nearest 10 and 100.
- Students can determine when rounding or estimating is more efficient during problem solving situations and justify their reasoning.

Sample Questions

- 1. What are 2 numbers, when added, their sum would round to 300?
- 2. Multiply 2 numbers and the product is almost 40. What 2 numbers could you have multiplied?
- 3. On Monday, 156 pennies were collected for the penny challenge. On Tuesday, 345 pennies were collected. Marisa says about 400 pennies were collected on Monday and Tuesday. Michael says about 500 pennies were collected on Monday and Tuesday. Use what you know about rounding to explain why you agree with Marisa or Michael.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Prior to implementing rules for rounding students need to have opportunities to investigate place value. A strong understanding of place value is essential for the continued development of number sense and the subsequent work that involves rounding numbers.

Building on previous understandings of the place value of digits in multi-digit numbers, place value is used to round whole numbers. Dependence on learning rules can be eliminated with strategies such as the use of a number line to determine which multiple of 10 or 100, a number is nearest. The number line can also be used to illustrate that 25, for example, is in the middle of 20 and 30. It is 5 away from 20 and 5 away from 30 and could therefore round to either 20 or 30. We decided as a rule that the number 25 would round to 30. Using the number line can help to illustrate this accepted rule. Students should be able to communicate that there are 5 numbers that round to the same decade and five numbers that round to the next decade. For example, 40, 41, 42, 43, and 44 round to the same decade (40). 65, 66, 67, 68, and 69 rounds to the next decade (70). As students' understanding of place value increases, the strategies for rounding are valuable for estimating, justifying and predicting the reasonableness of solutions in problem solving. Continue to use manipulatives like 100 charts and number lines as students work with rounding to different place values. Students should frequently engage in rich math discussions with the sharing out of strategies to extend on mathematical ideas.

Connections Across Standards

Solve two-step word problems and assess the reasonableness of answers using mental computation and estimation strategies (3.OA.8). Identify arithmetic patterns, and explain them using properties of operations (3.OA.9).

Solve word problems with adding and subtracting money and measurement within 1,000 (3.MD.1-2). Represent and interpret data (3.MD.3).

2.NBT.3 (Prior Grade Standard)

Read and write numbers to 1,000 using base-ten numerals, number names, expanded form G , and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc.

4.NBT.3 (Future Grade Standard)

Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.



3.NBT.2

Fluently add and subtract within 1,000 using strategies and algorithms ^G based on place value, properties of operations, and/or the relationship between addition and subtraction.

Essential Understandings

- Fluency is being efficient, accurate, and flexible with addition and subtraction strategies.
- Use place value understanding, properties of operations, and the relationships between operations to perform multi-digit arithmetic.

Common Misconceptions

Students may not have a conceptual understanding of place value so they would think 234 is the same as 2 + 3 + 4 rather than 200 + 30 + 4. Students may not have an understanding of 0 as a place holder.

Students may not have a conceptual understanding of place value so they would think 561 - 147 = 426, because they subtract the 7 in 147 from the 1 in 561 instead of regrouping.

Academic Vocabulary/Language

- associative property of addition.
- commutative property of addition
- identity property of addition
- digit
- algorithm

Tier 2

solve

Learning Targets

I can apply my understanding of place value to help solve addition and subtraction problems using various strategies.

I can apply the properties of operations to add numbers up to the thousands place value.

I can apply the relationship between addition and subtraction to subtract numbers to the thousands place value.

I can solve addition and subtraction problems within 1,000 using strategies and explain my thinking.

I can use a model to represent my strategy.

Assessment of Learning

- Students can demonstrate an understanding of place value to solve arithmetic problems.
- Students can fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (i.e. estimate sums, partial sums, partial differences, base ten models, number lines, count up, and count back).

Sample Questions

- 1. Solve 223 + 386 using 3 different strategies.
- 2. Create an addition sentence involving two 3-digit numbers whose sum is greater than 2,000 but less than 3,500.
- 3. Add a number to 361 that will increase the hundreds digit by 3, the tens digit by 2 and not changing the ones digit.
- 4. Vinnie accidentally added 235 to a number and got 537 when he was supposed to subtract 235. What should the answer be?
- 5. Jocelyn and James have a marble collection. Jocelyn has collected 775 marbles and James has collected 125. How many marbles have Jocelyn and James collected in all? Solve this problem using 2 different strategies.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students have been using a variety of strategies to add and subtract two-digit numbers. Students should now be given chances to see how those same strategies can be applied to fluently add and subtract bigger whole numbers within 1,000. Students should frequently be asked to estimate a sum or difference before calculating. They should also be asked to compare their results to estimates to determine if their work is reasonable. These strategies should be discussed so that students can make comparisons and move toward efficient methods. Number Talks are a great way to show a variety of strategies for this type of addition and subtraction. Students should be shown the traditional algorithm as a possible strategy to use. Their goal is to use strategies or algorithms in order to add and subtract within 1,000 efficiently, accurately and flexibly.

Connections Across Standards

Solve two-step word problems and assess the reasonableness of answers using mental computation and estimation strategies (3.OA.8). Identify arithmetic patterns, and explain them using properties of operations (3.OA.9).

Solve word problems with adding and subtracting money and measurement within 1,000 (3.MD.1-2).

Represent and interpret data (3.MD.3).

2.NBT.5 (Prior Grade Standard)

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT.4 (Future Grade Standard)

Fluently $^{\rm G}$ add and subtract multi-digit whole numbers using a standard algorithm $^{\rm G}$.



3.NBT.3

Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90, e.g., 9×80 , 5×60 using strategies based on place value and properties of operations.

Essential Understandings

- Use place value understanding and properties of operations to multiply a one-digit number by multiples of 10.
- Understand that zero is a placeholder.

Common Misconceptions

A problem like 5×40 is critical because of the discussion that would follow this problem. Reasoning for this problem would be that it represents 5 groups of 4 tens. When multiplying 5 groups of 4, you get the answer of 20. This may lead to confusion for some students because the product of the single digit number already ends in zero. Be sure to go back to the place value language. 5 groups of 4 is 20 therefore, 5 groups of 4 tens would be 20 tens. 20 tens is the same as 200. Avoid teaching tricks such as "add the zeros at the end" and instead focus on what happens as we move a digit to the left in our place value system (e.g. 4, 40, 400, 4,000). For true understanding students need to understand and be able to explain the place value reasoning.

Academic Vocabulary/Language

- multiple
- place value
- pattern
- product

Tier 2

explain

I can apply my understanding of place value to help solve arithmetic problems in various ways and explain my thinking. I can multiply a one-digit number by 10, 20, 30, 40, 50, 60, 70, 80, 90.

I can apply my understanding of place value and the properties of operations to multiply multiples of 10 (e.g., $4 \times 60 = 4 \times (6 \times 10) = (4 \times 6) \times 10$; or $4 \times 60 = (4 \times 30) + (4 \times 30)$.

- Students can apply understanding of place value to solve arithmetic problems in various ways.
- Students can identify patterns between basic facts and related multiplication situations.
- Students can explain the relationship when a number is multiplied by 10. (This understanding is critical as multiplying by 10 is more than simply adding a zero).
- Students can use concrete models or drawings to represent multiplication of a one-digit whole number by a multiple of 10.

Sample Questions

- 1. How would adding a 0 to the end of a number affect the value of the digits?
- 2. How are the products of 3×3 and 3×30 similar? How are they different?
- 3. How does multiplying 10×4 help you to solve 9×4 ? 20×2 ?
- 4. Create an equation using a one-digit whole number and a multiple of ten to equal 810.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Understanding what each number in a multiplication expression represents is important, for example 8×3 represents 8 groups of 3. Multiplication problems need to be modeled with pictures, diagrams or concrete materials to help students understand what the factors and products represent. The effect of multiplying numbers needs to be examined and understood.

The use of daily Number Talks can help students to understand what happens when we multiply a number by a multiple of 10. Problems in the Number Talks will lead to conversations about our place value system. Work students through a series of problems such as 4×1 , 4×10 and then 4×100 . Do the students see a pattern? What is happening as we add the zero? You can also explore a problem such as 3×40 . This problem represents 3 groups of 4 tens, which is the same as 12 tens which equals 120. Multiplying numbers by 10 is a foundational understanding for multiplying by multiples of ten. Students should be able to transfer this understanding to other digits times 10 and confidently explain what is happening mathematically.

Connections Across Standards

Solve two-step word problems and assess the reasonableness of answers using mental computation and estimation strategies (3.OA.8). Identify arithmetic patterns, and explain them using properties of operations (3.OA.9).

Solve word problems with adding and subtracting money and measurement within 1,000 (3.MD.1-2).

Represent and interpret data (3.MD.3).

2.NBT.8 (Prior Grade Standard)

Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.

4.NBT.5 (Future Grade Standard)

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.



3.NF.1

Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

Essential Understandings

- A fraction is a number showing a relationship between the parts and the whole
- Fractional parts have names that tell how many parts of a size are needed to make the whole (3 parts thirds; 4 parts fourths, etc.).
- Fractional parts can be described with words and symbols.

Common Misconceptions

Students may not understand that fractional parts are <u>equal</u> parts. In order to be thirds, for example, there can't just be 3 pieces, there have to be 3 <u>equal</u> pieces.

Students may be confused by the idea that the denominator (the bottom number) represents how many equal pieces are in the whole or set and the numerator (the top number) represents how many of those equal pieces you have.

Academic Vocabulary/Language

- numerator
- denominator
- fraction
- unit fraction
- whole
- equal parts

Tier 2

- explain
- represent

Learning Targets

I can explain any unit fraction as one part of a whole.

I can explain any fraction $(\frac{a}{b})$ as "a" (numerator) being the numbers of parts and "b" (denominator) as the total number of equal parts in the whole.

I can represent a fraction using a model and explain my representation.

I can compare representations of the same fraction and explain how they are related.

- Students can explain any unit fraction as one part of a whole.
- Students can represent a fraction and explain how various representations are related.

Sample Questions

- 1. Explain what Pedro means when he says that he has divided the shape into thirds.
- 2. Considering the unit fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$, describe how the denominator changing affects the value of the fraction. Use models and/or drawings to support your answer.
- 3. Use pattern blocks and have students identify what each shape represents if the whole is a hexagon. What does each pattern block represent if the trapezoid is the whole? If the rhombus is the whole? If the triangle is the whole?

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

In grades 1 and 2, students partitioned shapes into two, three, and four equal shares. They described those parts using fraction language of halves, thirds, and fourths. In Grade 3, the learning shifts to developing a more formal understanding of equal parts including the usage of mathematical symbols and number line diagrams. The instructional focus is on exploring the meaning and relationships in fractions; the significance of the whole; the unit fraction; and the initial understanding of equivalence of fractions using models and comparison of fractions (with like numerators or like denominators). In future grades, students will expand their understanding of fractions to formally solve problems with unlike denominators.

This is the initial experience students will have with fractions as numbers rather than just rectangles and circles partitioned into parts and is best done over time. Students need many opportunities to discuss fractional parts using concrete models to develop familiarity and understanding of fractions. In third grade, students work with equal parts of 2, 3, 4, 6, and 8. Students should be able to represent any fraction in multiple ways.

Connections Across Standards

Identify and use multiplicative relationships between numbers (3.OA.2, 3, and 9).

Measure with a ruler marked in halves and fourths of an inch (3.MD.4).

Connect the use of area models for multiplication to fractions (3.MD.7).

2.G.2-3 (Prior Grade Standard)

- **G.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
- **G.3** Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

4.NF.1 (Future Grade Standard)

Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.



3.NF.2

Learning Targets

Understand a fraction as a number on the number line; represent fractions on a number line diagram ^G.

- a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.
- b. Represent a fraction $\frac{a}{b}$ (which may be greater than 1) on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Essential Understandings

- Fractions can be represented with visual models such as rectangular area models, arrays, and length models including number lines.
- On a number line, the size of the part is measured by the distance from zero to the numbered point.
- A unit fraction represents one piece of the equal-sized pieces that make a whole $(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8})$.
- A unit fraction is the building block for fractions just as 1 is the building block for whole numbers.

Common Misconceptions

Students only think of fractions as a rectangle or circle partitioned into equal parts rather than as numbers at distinct points on the number line

Students may not understand that you count fractions just like you count whole numbers and that the size of the piece doesn't change as you count them. Therefore, when we count fourths, we count $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$ and so on.

The unit fraction represents the size of the pieces you are counting.

Academic Vocabulary/Language

- numerator
- denominator
- fraction
- unit fractions
- number line
- interval
- equal

Tier 2

- represent
- recognize
- explain
- label
- create

I can relate fractions to whole numbers.

I can describe how arithmetic with fractions is related (similar) to arithmetic with whole numbers.

I can label a number line using fractions.

I can create a number line with even intervals representing fractions.

I can determine where a fraction is located on a number line by partitioning and explain my reasoning.

- Students can explain the relationship between how arithmetic with fractions is related to arithmetic with whole numbers.
- Students can create, label, and represent fractions on a number number line.
- Students can determine where a fraction is located on a number line by partitioning and explain their reasoning.

Sample Questions

- 1. Mark the number line shown into fourths and label the mark that represents $\frac{3}{4}$.
- 2. Which letter represents the fraction $\frac{2}{3}$ on the number line shown?
- 3. Give students two number lines, one with endpoints of 0 and 1 and the other with endpoints marked 0 and $\frac{1}{2}$. Ask students to place $\frac{1}{4}$ on both lines. Explain why $\frac{1}{4}$ is placed differently on each number line.

NOTE: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Third grade is the first time students begin to think of fractions as numbers on a number line. Use their previous experiences with partitioning rectangles by showing that the distance between two numbers on a number line, the "whole", can be shaped into a rectangle too. Model how to break that rectangle into the designated number of equal parts. Model how many equal parts it takes to make that whole.

Understanding that a fraction is a quantity formed by part of a whole is essential to number sense with fractions. Fractional parts and unit fractions are the building blocks for all fraction concepts. Students need to relate dividing a shape into equal parts and representing this relationship on a number line, where the equal parts are between two whole numbers. Help students plot fractions on a number line, by using the meaning of the fraction. For example, to plot $\frac{4}{5}$ on a number line, there are 5 equal parts with 4 copies of the 5 equal parts. Use the number line to also show students that a whole number can be represented by a fraction. Model how four of the $\frac{1}{4}$ pieces is the same as the whole and therefore $\frac{4}{4}$ is equivalent to 1.

Fractions can have a value greater than one. These values can be represented on a number line as well. Third-grade students should be able to show fractions less than one and fractions greater than one on a number line and pictorial representation. As students practice exploring fractions greater that one, such as $\frac{5}{4}$, they may see the connection to $1\frac{1}{4}$. Conversions should not be explicitly taught.

Connections Across Standards

Identify and use multiplicative relationships between numbers (3.OA.2, 3, and 9).

Measure with a ruler marked in halves and fourths of an inch (3.MD.4).

Connect the use of area models for multiplication to fractions (3.MD.7).

2.G.3 (Prior Grade Standard)

Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

4.NF.6 (Future Grade Standard)

Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.



3.NF.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
- b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model ^G. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model

Essential Understandings

- Two fractions can be compared when the two fractions refer to the same whole.
- When comparing fractions with the same denominator, the fraction with the greater numerator is greater because more unit fractions are needed to make up the part.
- When comparing unit fractions with different denominators, the fraction with the larger denominator is smaller because it takes more equal sized pieces to make the whole.

Common Misconceptions

The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set can be very confusing for students. Until now, students have only compared whole numbers and have understood that the smaller a number, the less it is, or the larger a number, the more it is. The use of different models, such as fraction bars and number lines will allow students to begin to grasp this concept and help them to compare unit fractions and reason about their sizes

Academic Vocabulary/Language

- numerator
- denominator
- equivalent fraction
- fraction
- unit fractions
- number line

Tier 2

- explain
- reasoning
- recognize
- generate
- express
- compare
- record
- identify
- create

Learning Targets

I can explain, identify and create equivalent fractions and justify my thinking. I can identify and write a whole number as a fraction.

I can locate equivalent fractions on a number line and explain the relationship.

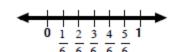
I can compare two fractions and record comparisons with the same numerator or the same denominator using >, =, or < using models or numbers and justify my answer.

Assessment of Learning

- Students can identify and represent whole numbers as fractions.
- Students can explain why equivalent fractions must describe the same-size whole.
- Students can compare and explain equivalent fractions with representations including color tiles, pattern blocks, Cuisenaire Rods, fraction tiles, and models.
- Students can record comparisons using <, >, or =.

Sample Questions

1. On the number line shown, label the places where $\frac{1}{3}$ and $\frac{2}{3}$ should appear.



2. Which two fractions does this figure show to be equivalent?



- 3. Show $\frac{1}{2}$ and $\frac{4}{8}$. Have students compare the two fractions using different representations. Ask students to explain why these two fractions are equal and have them brainstorm other equivalent fractions to $\frac{1}{2}$.
- 4. Marcus says $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent. Is Marcus correct? Explain your thinking using a model.

Note: Expectations in this domain are limited to fractions with denominators 2, 3, 4, 6 and 8.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Once students have a firm understanding of fractions as a/b, they can begin to compare them. Students must understand that fractions can be compared only if they refer to parts of the same size whole. Equivalent fractions can be recognized and generated using fraction models. Students should use different models and decide when to use a particular model. Students need to remember that when fractions are equivalent, the wholes are the same. If the wholes are different, the fractions are not equivalent. Cuisenaire Rods are a nice tool since they transfer easily to a number line. You could also use transparencies to show how equivalent fractions measure up on the number line. Benchmarks such as $0, \frac{1}{2}$ and 1 are helpful for third graders when comparing fractions. Venn diagrams are useful in helping students organize and compare fractions to determine the relative size of the fractions, such as more than $\frac{1}{2}$, exactly $\frac{1}{2}$ or less than $\frac{1}{2}$. Fraction bars showing the same sized whole can also be used as models to compare fractions. Students are to write the results of the comparisons with the symbols >, =, or < and justify the conclusions with a model.

Columbus City Schools 2020

Connections Across Standards

Identify and use multiplicative relationships between numbers (3.OA.2, 3, and 9).

Measure with a ruler marked in halves and fourths of an inch (3.MD.4).

Connect the use of area models for multiplication to fractions (3.MD.7).

2.G.3 (Prior Grade Standard)

Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

4.NF.5 (Future Grade Standard)

Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $\frac{3}{10}$ as $\frac{3}{100}$, and add $\frac{3}{10}+\frac{4}{100}=\frac{34}{100}$. In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.

Columbus City Schools 2020



3.MD.1

Work with time and money.

- a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock.
- b. Solve word problems by adding and subtracting within 1,000 dollars with dollars and cents (not using dollars and cents simultaneously) using the \$ and ¢ symbol appropriately (not including decimal notation).

Essential Understandings

- Time is measured in hours and minutes.
- Time can be measured to the nearest minute.
- Elapsed time measures the duration of an event.
- Money is added and subtracted using whole number strategies.
- The dollar symbol and cent symbol are not used simultaneously.

Common Misconceptions

Students need to understand that there are 60 minutes in an hour and that all 60 minutes are represented on a clock, not just the multiples of 5.

Students need to understand that 100 cents is the same as one dollar. Students need to use the \$ and $$\phi$ symbols correctly.

Academic Vocabulary/Language

- analog clock
- digital clock
- time interval
- number line diagram
- dollar
- cent
- elapsed time

Tier 2

- solve
- represent

I can relate my understanding of time to real world problems.

Learning Targets

I can tell and write time to the nearest minute and use that understanding when solving real world problems. I can add and subtract intervals of time using minutes using models to represent my thinking when solving real world problems.

I can add and subtract dollars and cents within 1,000 dollars by applying my understanding of addition and subtraction when solving real world problems.

- Students can say and write time to the nearest minute.
- Students can solve addition and subtraction word problems involving durations of time measured in minutes.
- Students can solve problems where they have to add and subtract dollars and cents within 1,000.

Sample Questions

- 1. Oliver has 2 quarters, 1 dime and 7 pennies in his piggy bank. How much does Oliver have in all? Justify your thinking.
- 2. Sally left for school at 7:45 a.m. Maria left at 8:05 a.m. How many minutes later did Maria leave than Sally?
- 3. Create a number line to find the difference between between 12:45 p.m. and 2:15 p.m.
- 4. Jose got to school at 8:25 a.m. It took Jose 35 minutes to get to school. What time did he leave for school? Draw two clocks to represent the time Jose arrived at school and the time he left for school.
- 5. The basketball game begins at 12:30 p.m. If the game lasts 1 hour and 15 minutes, when will it be over? Explain your thinking.
- 6. Explain how a number line can be used to show elapsed time.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

A clock is a common instrument for measuring time. Learning to tell time has much to do with learning to read a dial-type instrument and little with time measurement. Students have experience in telling and writing time from analog and digital clocks to the hour and half hour in Grade 1 and to the nearest five minutes, using a.m. and p.m. in Grade 2. Now students will tell and write time to the nearest minute and measure time intervals in minutes. Provide analog clocks that allow students to move the minute hand. Students need experience representing time from a digital clock to an analog clock and vice versa. Provide word problems involving addition and subtraction of time intervals in minutes. Have students represent the problem on a number line. Students should relate using the number line with subtraction from Grade 2. In Grade 3, students learn that elapsed time can be found by finding the total amount of time that passes between a starting time and an ending time. Students should become familiar with tools such as a clock, timeline, number sentence, and other tools to accurately calculate and represent elapsed time.

In Grade 2, students found the value of a collection of quarters, nickels, dimes, and pennies and added and subtracted dollars with dollars and cents with cents (not using dollars and cents simultaneously) within 100. In Grade 3, students add and subtract money (dollars with dollars and cents with cents, not using dollars and cents simultaneously) to solve word problems within 1,000. Students do not need to understand decimals and decimal notation in third grade.

Time and money word problems should represent all problem types. This includes start unknown, change unknown, and result unknown.

Connections Across Standards

Fluently add and subtract money within 1,000 (3.NBT.2).

Fluently multiply and divide and solve word problems within 100 with an emphasis on multiplication and division situations for measurement (3.OA.3, 7).

Solve word problems using the four operations (3.0A.8). Understand a fraction as a number on a number line (3.NF.2). Apply equivalency of fractions (3.NF.3).

2.MD.7 (Prior Grade Standard)

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

4.MD.2 (Future Grade Standard)

Solve real-world problems involving money, time, and metric measurement.

- a. Using models, add and subtract money and express the answer in decimal notation.
- b. Using number line diagrams ^G, clocks, or other models, add and subtract intervals of time in hours and minutes.
- c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.



3.MD.2

Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve

one step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Table 2.

Essential Understandings

- Mass is measured in kilograms or grams.
- Liquid volume is measured in liters.
- Mass and liquid volume word problems are solved using whole number strategies.

Common Misconceptions

Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams may be read as 81 grams. Students realize it is one away from 80, but do not think of it as 79 grams. Students may struggle to read liquid volume in a drawing of a beaker or other liquid measuring tool. Model for them how to read the line correctly.

Academic Vocabulary/Language

- capacity
- liquid volume
- liter (L)
- metric unit
- milliliter (mL)
- unit
- grams (g)
- kilograms (kg)
- mass

Tier 2

- estimate
- solve
- represent

Learning Targets

I can solve real world problems involving liquid volume and the mass of objects by applying my understanding of addition, subtraction, multiplication, and division.

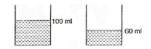
I can estimate and measure liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (L).

I can use drawings to solve one step word problems involving milliliters and liters, grams and kilograms to represent my thinking.

- Students will represent and solve one-step word problems involving masses or volumes using addition, subtraction, multiplication, and division.
- Students can use drawings to represent solutions to solve one step word problems involving milliliters and liters, grams and kilograms.

Sample Questions

1. How many milliliters are there when you combine the two containers?



- 2. Describe something you would measure in kilograms. Could it also be measured in liters? Explain your thinking.
- 3. Emily's fish tank holds 60 liters of water. She fills the tank by filling up a 6 liter container with water and dumping it in the tank. How many times will she fill up the container with water for the fish tank? Justify your answer.
- 4. A gallon of punch holds 16 cups of punch. The third-grade class is having a party. Each student drinks 2 cups of punch. There are 12 students attending the party. How much punch should the teacher buy? Explain your answer.

Note: Excludes compound units such as cm³ and finding the geometric volume of a container. Excludes multiplicative comparison problems (problems involving notions of "times as much"; See Glossary Table 2

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Provide opportunities for students to use appropriate tools to measure and estimate liquid volumes in liters and milliliters and masses of objects in grams and kilograms. Liquid measure is measured in liters, ounces, gallons, or quarts. Mass is a measure of the amount of how heavy an object is and is measured using grams or kilograms. The standard is specific to grams, kilograms, and liters. However, it is appropriate for students to work with standard measurement units including cups and gallons, as these are also measures of capacity. Students need practice in reading the scales on measuring tools since the markings may not always be in intervals of one. The scales may be marked in intervals of two, five or ten. Allow students to hold gram and kilogram weights in their hand to use as a benchmark. Use water colored with food coloring so that the water can be seen in a beaker. Students should estimate volumes and masses before actually finding the measuring. Show students a group containing the same kind of objects. Then, show them one of the objects and tell them its weight. Fill a container with more objects and ask students to estimate the weight of the objects. Use similar strategies with liquid measures. Be sure that students have opportunities to pour liquids into different size containers to see how much liquid will be in certain whole liters. Show students containers and ask, "How many liters do you think will fill the container?"

UNKNOWN PRODUCT		GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?	If 18 plums are to be packed 6 to a bag, then how many bags are needed?
	Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS ² , AREA ³	There are 3 rows of apples with 6 apples in each row. How many apples are there?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?
	Area example. What is the area of a 3 cm by 6 cm rectangle?	Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?
	Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Connections Across Standards

Fluently add and subtract money within 1,000 (3.NBT.2).

Fluently multiply and divide and solve word problems within 100 with an emphasis on multiplication and division situations for measurement (3.OA.3, 7).

Solve word problems using the four operations (3.0A.8).

Understand a fraction as a number on a number line (3.NF.2).

Apply equivalency of fractions (3.NF.3).

2.MD.1(Prior Grade Standard)

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

4.MD.1 (Future Grade Standard)

Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a



3.MD.3

Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more"

and "how many less" problems using information presented in the scaled graphs. For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.

Essential Understandings

- The key of a picture graph tells how many items each picture or symbol represents.
- A scaled graph (bar graph or line plot) is labeled using equal-sized intervals along the axes.
- The scale of a bar graph varies depending on the data set.

Common Misconceptions

Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip-counting when determining the value of a bar since the scale is not in single units.

Students may think the scale of a bar graph should always have an interval of 1. Experiences with large numbers as data can help students to choose the best interval to display the data clearly.

Academic Vocabulary/Language

- picture graph
- scale
- interval
- bar graph
- key
- data
- table

Tier 2

- represent
- solve
- present
- analyze
- interpret

Learning Targets

I can create a scaled picture graph to represent information and interpret the data.

I can create a scaled bar graph to represent information and interpret the data.

I can answer one and two step questions about a picture graph or bar graph and justify my thinking.

- Students can read and interpret scaled bar graphs in order to solve one and two-step "how many more" and "how many less" problems.
- Students can represent data using a scaled picture graph or bar graph.

Sample Questions

- 1. A bar graph shows the greatest data point at 55. The lowest data point is 22. How many more votes is the greatest number than the lowest?
- 2. Provide a set of data to students that describes the ages of a group of people at a family gathering. Have students create a graph to represent the data.
- 3. Draw a bar graph with the data shown.

Money Donated for Charity

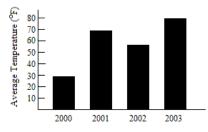
Monday - \$25

Tuesday - \$12

Wednesday - \$5

Thursday - \$22

4. What year had the highest average temperature?



Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs that may have an interval greater than one. Intervals for the graphs should relate to multiplication and division with 100 (product is 100 or less and numbers used in division are 100 or less). Students should determine an appropriate scale for a set of data and should explore and record data in several categories in the bar graph. In picture graphs, use values for the icons in which students are having difficulty with multiplication facts. For example, \square represents 7 people. If there are three \square , students should use known facts to determine that the three icons represent 21 people. Students are to draw picture graphs in which a symbol or picture represents more than one object. Use symbols on picture graphs that students can easily represent half of, or know how many half of the symbol represents. Once the bar graph or picture graph are complete, ask questions that require students to compare quantities and use mathematical concepts and skills. Give students opportunities to interpret data to solve two-step problems using information displayed in a graph.

Connections Across Standards

Solve problems involving multiplication (3.0A.3).

Solve two-step problems using the four operations (3.OA.8).

Understand a fraction as a number on a number line (3.NF.2).

2.MD.10 (Prior Grade Standard)

Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put together, take-apart, and compare problems in a graph. See Table 1.

4.MD.4 (Future Grade Standard)

Display and interpret data in graphs (picture graphs, bar graphs, and line plots ^G) to solve problems using numbers and operations for this grade.



3.MD.4

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot ^G, where the horizontal scale is

marked off in appropriate units—whole numbers, halves, or quarters.

Essential Understandings

- Length measurement data can be generated and used to create a line plot.
- The scale of a line plot can be whole numbers, halves, or quarters.

Common Misconceptions

A line plot has data points marked above a number line. Students may incorrectly choose a line plot to display data such as favorite foods or class pets.

Students must try to keep the "x" marks on a line plot consistently sized and evenly spaced.

Academic Vocabulary/Language

- line plot
- halves
- fourths (quarter)
- data
- units
- intervals

Tier 2

- generate
- show
- plot

Learning Targets

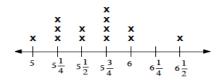
I can use a ruler to measure objects to the half inch and fourth (quarter) inch to generate data and create a line plot to represent the data.

I can interpret the data on a line plot and use that information to solve real world problems.

- Students can use a ruler to measure whole, half, and quarter inches.
- Students can record measurement data using whole, half, and quarter inches.
- Students can create a line plot marked off in whole numbers, half, or quarter units.

Student questions:

- 1. Kimani broke her ruler and lost one part of it. Her ruler now goes from 5 inches to 12 inches. Kimani told her teacher she could not use the ruler to measure her pencil. Is Kimani correct or incorrect? Explain why or why not. (*The purpose is for students to understand that a ruler measures the difference between 2 points. Students do not have to begin measuring with the zero. Students should also be able to explain how they determined the measurement of the pencil).*
- 2. Robin says that the string is 4 inches. David said that it was $8 \frac{1}{2}$ inches. Whose measurement is correct? Explain.
- 3. Measure the lengths of all of the pencils belonging to the students in your classroom to the nearest quarter of an inch. Create a line plot to display this data.
- 4. The graph shows my data after measuring 13 pencils. Give two observations about the data?



Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students are to measure lengths using rulers marked with halves and fourths (quarter) of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots.

Students should understand that a ruler measures the difference between 2 points. Students do not have to begin measuring with the zero.

Connections Across Standards

Solve problems involving multiplication (3.0A.3).

Solve two-step problems using the four operations (3.OA.8).

Understand a fraction as a number on a number line (3.NF.2).

2.MD.9 (Prior Grade Standard)

Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot ^G, where the horizontal scale is marked off in whole number units.

4.MD.4 (Future Grade Standard)

Display and interpret data in graphs (picture graphs, bar graphs, and line plots ^G) to solve problems using numbers and operations for this grade.



3.MD.5

Recognize area as an attribute of plane figures and understand concepts of area measurement.

- a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
- b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Essential Understandings

- Area is an attribute of plane figures that is measured using square units.
- Area is found by covering the inside of a two-dimensional plane figure with square units without gaps or overlaps, and then counting the number of square units used.

Common Misconceptions

Students may not completely cover a shape with unit squares but may instead only put squares around the border of the shape.

Students may not count all of the squares that cover the shape or may incorrectly count them.

Area and perimeter are often mixed up by third graders.

Academic Vocabulary/Language

- area
- square unit
- unit square
- plane figure

Tier 2

recognize

Learning Targets

I can use a unit square to measure area.

I can describe area as the measure of space within a plane figure.

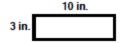
I can count unit squares to measure the area of a plane figure.

I can explain why area is measured in square units.

- Students can describe a square unit.
- Students have an understanding that area is the measure of space within a plane figure.
- Students can explain why area is measured in square units.

Sample Questions

1. What type of units would you use to completely cover the shape shown? Explain your thinking.



- 2. Choose a pattern block. Create a shape made up of 30 of those blocks. Now choose another block. How many blocks of this type will you need to make a shape that takes up the same space?
- 3. Based on your understanding, how would you describe the definition of area to a classmate who was absent during the lesson?
- 4. Students should have practice discovering area by using tiles or square units to cover objects to find the area.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

This standard introduces the formal study of area. In previous grades, students used linear measurement with no gaps or overlaps. In Grade 2, students used addition to find the total number of objects arranged in rectangular arrays (up to 5×5). Now in Grade 3, students measure area in square units. They first count unit squares in tilings (concrete) and later use multiplication and addition of whole numbers to find areas of figures composed of rectangles in real-world situations. Students can cover rectangular shapes with tiles and count the number of units (tiles) to begin developing the idea that area is a measure of covering. Area describes the size of an object that is two-dimensional. The formulas should not be introduced before students discover the meaning of area.

Connections Across Standards

Solve problems involving multiplication (3.0A.3).

Solve two-step problems using the four operations (3.OA.8).

Apply properties of operations (3.OA.5).

Partition shapes into parts with equal areas (3.G.2).

Determine the unknown whole number in a multiplication of division equation (3.OA.4).

2.G.2 (Prior Grade Standard)

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them

4.MD.3 (Future Grade Standard)

Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.



3.MD.6

Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Common Misconceptions

Students may not completely cover a shape with unit squares but may instead only put squares around the border of the shape. Students may not count all of the

Students may not count all of the squares that cover the shape or may incorrectly count them.

Area and perimeter are often mixed up by third graders.

Academic Vocabulary/Language

- square unit
- unit square
- area
- plane figure
- square cm
- square m
- square in
- square ft

Essential Understandings

- Area is an attribute of plane figures that is measured using square units.
- Area is found by covering the inside of a two-dimensional plane figure with square units without gaps or overlaps, and then counting the number of square units used.

Learning Targets

I can model measuring an area by using unit squares.

I can count unit squares to measure the area of a plane figure.

- Students can measure the area of a shape by covering it with square units and counting the number of unit squares used.
- Students will use tiling (without gaps or overlaps) to find the area of a rectangle by counting unit squares.
- Students will use appropriate units (square cm, square m, square in, square ft, and improvised units).

Sample Questions

1. After drawing the unit squares that would completely cover the shape shown, determine the area.



- 2. Andrea wants to create a 4-sided shape with the largest area possible. She has a choice of sides that are 1 unit, 3 units, 4 units, or 5 units. What measurements should Andrea use? Create a pictorial representation to justify your solution.
- 3. Shayna has a shape with 3 units in the first row. If she has 7 rows under the first row with 3 units in each, what is the area of her shape?
- 4. Provide students with various shaded in figures on a grid. Have students determine the areas and determine the figure with the greatest area.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

The area of a rectangle can be determined by having students lay out unit squares and count how many square units it takes to cover the rectangle completely without overlaps or gaps. Students need to develop the meaning for computing the area of a rectangle. A connection needs to be made between the number of squares it takes to cover the rectangle and the dimensions of the rectangle.

Ask questions such as:

- What does the length of a rectangle describe about the squares covering it?
- What does the width of a rectangle describe about the squares covering it?

Questions like these will help students apply strategies such as counting the squares, adding the rows/columns of shaded squares, and tiling. These strategies will help students discover that the area is the same as it would be by multiplying the side lengths. Only whole number side lengths should be used.

Connections Across Standards

Solve problems involving multiplication (3.0A.3).

Solve two-step problems using the four operations (3.OA.8).

Apply properties of operations (3.OA.5).

Partition shapes into parts with equal areas (3.G.2).

Determine the unknown whole number in a multiplication of division equation (3.OA.4).

2.G.2 (Prior Grade Standard)

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

4.MD.3 (Future Grade Standard)

Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.



3.MD.7

Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

- b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole number products as rectangular areas in mathematical reasoning.
- c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and b + c is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model).
- d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Essential Understandings

- The area of a rectangle can be found by multiplying the lengths of two adjacent sides of the rectangle.
- The area of a rectangle can be found by being decomposed into two rectangular parts; finding the areas of the two smaller rectangles; and then adding the two smaller areas to find the total area.
- A figure composed of rectangles may be decomposed into rectangles whose areas may be added to find the area of the figure.

Common Misconceptions

Students often find area when asked for perimeter and vice versa.

Students may not use their understanding of the characteristics of quadrilaterals when they decompose a compound figure and try to find the new side lengths.

Students may find the area of each rectangle after they

decompose a compound shape but then do not add the areas together to find the total area.

Academic Vocabulary/Language

- area
- area model
- square unit
- unit square
- formula
- decompose
- distributive property

Tier 2

- relate
- represent
- show
- model
- recognize

I can construct a model to show the area of a rectangle by using tiles.

I can apply my understanding of multiplication when finding the area of a rectangle by multiplying the length and the width.

I can construct a model to show the area of a rectangle using tiles when the rectangle is divided into two rectangles.

I can find the area of a large rectangle by dividing it into smaller rectangles and adding their areas.

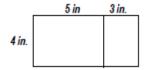
I can solve area word problems and explain my thinking.

Learning Target

- Students can explain the relationship between tiling and multiplying side lengths to find the area of rectangles.
- Students use area models to explain the distributive property.
- Students can apply knowledge of area to solve word problems by multiplying.

Sample Questions

- 1. Show all of the possible rectangular arrays for the number 12.
- 2. An object has an area of 48 square centimeters. What could the length and width be?
- 3. Mrs. Reed gave each student two pieces of paper. One measured 4 inches by 5 inches and the other 4 inches by 3 inches. Students were told to tape them together as shown below. Find two different ways to calculate the total area of the paper and explain why it works.



4. Tiffany's bedroom is in the shape of a rectangle with side lengths of 10 feet and 17 feet. She wanted to calculate the area by multiplying the side lengths, but she thinks it would be easier to break the 17 into 10 and 7 before multiplying. Can Tiffany do this? Explain why this strategy would or would not work.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

The concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column. Ask questions about the dimensions if students do not make these discoveries.

For example:

- How do the squares covering a rectangle compare to an array?
- How is multiplication used to count the number of objects in an array?



Students should also make the connection of the area of a rectangle to the area model used to represent multiplication. This connection justifies the formula for the area of a rectangle.

Provide students with the area of a rectangle (for example, 16 square inches) and have them determine possible lengths and widths of the rectangle. Expect different lengths and widths such as, 8 inches by 2 inches or 4 inches by 4 inches.

With practice, students can build a shape with a given area and explain using pictures, numbers or words how they built their shape. Students may represent their thinking using manipulatives such as color tiles, unifix cubes, or pictorial representation (draw square tiles). Students should also have the opportunity to solve for area by using the distributive property. Provide students many opportunities to decompose area with simple rectangles before advancing to more complicated figures.

Connections Across Standards

Solve problems involving multiplication (3.0A.3).

Solve two-step problems using the four operations (3.OA.8).

Apply properties of operations (3.OA.5).

Partition shapes into parts with equal areas (3.G.2).

Determine the unknown whole number in a multiplication of division equation (3.OA.4).

2.G.2 (Prior Grade Standard)

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

4.MD.3 (Future Grade Standard)

Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.



3.MD.8

Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and

exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Essential Understandings

- Perimeter is found by adding all the outside (exterior) side lengths of a polygon.
- An unknown side length of a polygon can be found when given the perimeter and other side lengths or properties of the polygon.
- Different rectangles may have the same perimeter but different areas. Different rectangles may have the same area but different perimeters.

Common Misconceptions

Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter. Encourage students to use their understanding of the characteristics of polygons (e.g., opposite sides of a rectangle are equal) in order to label the missing side lengths and add all of the side lengths to find the perimeter.

Academic Vocabulary/Language

- area
- square unit
- unit square
- perimeter
- polygons

Learning Targets

I can apply strategies of addition when finding the perimeter of a shape given all side lengths.

I can relate strategies of addition when finding the perimeter of a shape with an unknown side length.

I can determine and explain how two rectangles can have the same perimeters and different areas.

I can solve perimeter word problems and explain my thinking.

- Students can describe that the perimeter is the distance around a figure or shape.
- Students can determine the perimeter of polygons when given the lengths of all sides.
- Students can find the unknown side lengths of polygons when given the perimeter.
- Students can apply strategies to solve word problems that explain how rectangles with the same perimeter can have different areas and how rectangles with the same area can have different perimeters.
- Students will be provided multiple opportunities to discover the relationship between area and perimeter through exploratory tasks and class discussions.

Sample Questions

1. The perimeter of this polygon is 28 in. What is the length of side x? Explain your thinking.



- 2. Draw two different rectangles so both have a perimeter of 24 feet but their areas are different.
- 3. The perimeter of a rectangular chalkboard is 45 ft. The top of the chalkboard is 15 ft. What are the lengths of the other 3 sides?
- 4. The construction crew was building a playground with a perimeter of 96 feet. The students were trying to figure out what the playground was going to look like. The playground was not in the shape of a square and the longest side was 32 feet long. What could the playground look like and what could each of the measurements be?

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

Students need a lot of practice finding the perimeter of polygons when all side lengths are given. Students also need to find the perimeter when all side lengths are not given. Students should use their understanding of the characteristics of polygons to find the missing side lengths. For example, give students a square where only one side length is given. Ask students what they know about squares and how that can help them to find the missing side lengths and then the perimeter. Geoboards can be used to find the perimeter of rectangles also. Provide students with different perimeters and have them create the rectangles on the geoboards. Have students share their rectangles with the class. Have discussions about how different rectangles can have the same perimeter with different side lengths. Geoboards are also a good tool for modeling how rectangles can have the same perimeter but different areas or the same area but different perimeters.

Once students know how to find the perimeter of a rectangle, they can find the perimeter of rectangular-shaped objects in their environment. They can use appropriate measuring tools to find lengths of rectangular-shaped objects in the classroom. Present problem situations involving perimeter, such as finding the amount of fencing needed to enclose a rectangular shaped park, or how much ribbon is needed to decorate the edges of a picture frame. Also present problem situations in which the perimeter and two or three of the side lengths are known, requiring students to find the unknown side length.

Connections Across Standards

Distinguish between area and perimeter (3.MD.5-7).

Solve two-step problems using the four operations (3.OA.8).

Apply properties of operations (3.OA.5).

Fluently add and subtract within 1,000 (3.NBT.2).

Determine the unknown whole number in an expression (3.OA.4).

2.MD.3 (Prior Grade Standard)

Estimate lengths using units of inches, feet, centimeters, and meters.

4.MD.3 (Future Grade Standard)

Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.



3.G.1

Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).

Essential Understandings

- Polygons are closed two-dimensional shapes with straight sides.
- Polygons can be described by the number of sides.
- Polygons can be described by the presence or absence of square corners/right angles.

Common Misconceptions

Students may identify a square as a "non-rectangle" or a "non-rhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well. Using straws or Anglegs to make two congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

Academic Vocabulary/Language

- attribute
- hexagon
- octagon
- pentagon
- polygon
- quadrilateral
- triangle
- rhombus
- rectangle
- square
- pentagon
- right angle

Tier 2

- categorize
- recognize
- draw

Learning Targets

I can define quadrilaterals and provide examples.

I can identify and draw quadrilaterals and explain my thinking.

I can classify shapes into groups based on their properties, such as polygons or quadrilaterals and justify my reasoning.

I can explain how a rhombus, a rectangle and a square are alike and different.

- Students can identify, describe, define, and sort shapes by their attributes.
- Students can apply what they know about quadrilaterals to identify and draw quadrilaterals.
- Students will explore classifying triangles, quadrilaterals (rhombuses, rectangles, and squares) and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).

Sample Questions

- 1. A certain shape is similar to a rectangle, but it is not a rectangle. What could the shape be and why? Explain how the shapes are similar and how they are different.
- 2. Provide students the opportunity to discover quadrilaterals by their attributes. (e.g. Draw a quadrilateral that has exactly two right angles and no sides are the same length. Compare your share with your neighbor's shape. Does your shape have a name? Explain your reasoning.

Ohio Department of Education Model Curriculum Instructional Strategies and Resources

The focus now is on identifying and describing properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. These properties allow for generalizations of all shapes that fit a particular classification. Development in focusing on the identification and description of shapes' properties should include examples and non-examples, as well as examples and non-examples drawn by students of shapes in a particular category. For example, students could start with identifying shapes with right angles. An explanation as to why the remaining shapes do not fit this category should be discussed. Students should determine common characteristics of the remaining shapes. This standard also includes a lot of specific academic vocabulary that is new to third graders so time should be spent developing definitions for words such as polygon, quadrilateral, congruent, vertices, right angle, etc. Students should be encouraged to use proper vocabulary when discussing shapes.

Students should not be expected to memorize a formal definition of the quadrilaterals but be able to identify them and name them.

Connections Across Standards

Understand that a unit fraction is the quantity formed when a whole is partitioned into equal parts and one part is selected (3.NF.1).

2.G.1 (Prior Grade Standard)

Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders.

4.G.2 (Future Grade Standard)

Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

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3.G.2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and

describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Essential Understandings

- Some shapes can be partitioned into parts with equal areas.
- When shapes are partitioned into equal areas, the area of each part is the unit fraction of the whole.

Common Misconceptions

Students may draw lines on a shape to partition it into parts, but those parts may not be equal. Just because a shape has been partitioned into 3 parts it does not mean that those parts represent thirds.

Academic Vocabulary/Language

- partition
- unit fraction
- whole
- area
- equal

Tier 2

- express
- describe

Learning Targets

I can construct a model to partition a shape into equal parts.

I can apply my understanding of a unit fraction to label the area of each part as a fraction.

Assessment of Learning				
 Students can partition shapes into equal parts understanding that the parts have equal areas. 				
• Students can express the area of each part as a fraction.				
• Students can write a unit-fraction for partitioned shapes.				
Sample Questions				
1. Create a design in which a square represents $\frac{1}{4}$ of the area of the design.				
2 Partition the chang below into eight equal parts and label each part	with the correct fraction that describes each part. How do you know you			
2. Partition the shape below into eight equal parts and label each part with the correct fraction that describes each part. How do you know yo are correct?				
are correct?				
3. Give two examples of how or when you partition shapes into equal areas in everyday life.				
Ohio Department of Education Model Curriculum Instructional Strategies and Resources				
In Grade 2, students partitioned rectangles into two, three or four equal shares, recognizing that the equal shares need not have the same shape.				
They described the shares using words such as halves, thirds, half of, a third of, etc. and described the whole as two halves, three thirds or four				
fourths. Students must understand that fractional parts are equal parts, so just partitioning a shape into parts does not make it fractional parts. Have				
students draw different shapes and see how many ways they can partition shapes into parts with equal areas. This standard focuses on unit fractions				
with denominators of 2, 3, 4, 6, and 8.				
Once students show understanding of partitioning they should connect their	r recordings to the symbolic recording of a fraction explaining the			
number of partitions and the number of parts identified.				
Connections Across Standards				
Understand that a unit fraction is the quantity formed when a whole is partitioned into equal parts and one part is selected (3.NF.1).				
2.G.3 (Prior Grade Standard)	4.G.3 (Future Grade Standard)			
Partition circles and rectangles into two, three, or four equal shares;	N/A			
describe the shares using the words halves, thirds, or fourths and				
quarters, and use the phrases half of, third of, or fourth of and quarter of.				
Describe the whole as two halves, three thirds, or four fourths in				
real-world contexts. Recognize that equal shares of identical wholes need				
not have the same shape.				